# Modified Fincke-Pohst Algorithm for Low-Complexity Iterative Decoding over Multiple Antenna Channels

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*Abstract* — **In recent years, soft iterative decoding techniques have been shown to greatly improve the bit error rate performance of various communication systems. For multiple antenna systems employing space-time codes, however, it is not clear what is the best way to obtain the soft-information required of the iterative scheme with low complexity. In this paper, we propose a modification of the Fincke-Pohst (sphere decoding) algorithm [1] to estimate the maximum a posteriori (MAP) probability of the received symbol sequence. The new algorithm (FP-MAP) solves a nonlinear integer least-squares problem and, over a wide range of rates and SNRs, has polynomial-time (often cubic) expected complexity. The FP-MAP algorithm provides soft detection information for the soft channel decoder. The soft decoder's output is then fed back to the FP-MAP, and iterated on. The performance of the FP-MAP algorithm on a multiple antenna system employing turbo code is demonstrated.**

## I. Introduction

We assume a discrete-time block fading multiple antenna channel model with $M$ transmit and $N$ receive antennas, where the transmitted signal $\mathbf{s}$ and received signal $\mathbf{x}$ are related by

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{v},$$

where $\mathbf{H}$ is the channel matrix, and $\mathbf{v}$ is the additive noise vector. Furthermore, transmitted symbols $\mathbf{s}$ are obtained upon modulating a coded sequence $\{c_i\}$ onto an $m$-dimensional ($m = 2M$) subset of a lattice $\mathcal{D}_L^m$ (lattice subset $\mathcal{D}_L^m$ spans over $L$ points in each of $m$ dimensions).

For a known channel in AWGN, MAP detection is equivalent to the optimization problem

$$\min_{s\in\mathcal{D}_L^m} \left[ \|x - Hs\|^2 - \sum_{k=1}^{m} \log p(s_k) \right], \tag{1}$$

where $p(s_k)$ denotes a priori probability of a $k^{th}$ entry in a symbol vector. Applying the idea of Fincke-Pohst algorithm (which has polynomial expected complexity [2]), rather than to search over the entire lattice, we search only over lattice points $\mathbf{s}$ that belong to the geometric body described by

$$r^2 \geq (\mathbf{s} - \hat{\mathbf{s}})^* R^* R(\mathbf{s} - \hat{\mathbf{s}}) - \sum_{k=1}^{m} \log p(s_k), \tag{2}$$

where $R$ is lower triangular matrix following QR factorization of $H$ and $\hat{\mathbf{s}}$ is standard least-squares estimate. [Note that, unlike in the original sphere decoder algorithm, this geometric body is no longer a hypersphere.] The search radius $r$ in (2) can be chosen according to the statistical properties of the noise and a priori distribution of $\mathbf{s}$.

For an iterative decoding scheme, we require soft information, i.e., probability that each bit is decoded correctly. To this end, consider log-likelihood ratio of the form

$$L(c_i|\mathbf{x}) = \log \frac{\sum_{s\,:\,c_i\,=\,+1} e^{-\|x-Hs\|^2+\sum_j \log p[s_j]}}{\sum_{s\,:\,c_i\,=\,-1} e^{-\|\mathbf{x}-H\mathbf{s}\|^2+\sum_j \log p[s_j]}} \tag{3}$$

Computing (3) over entire signal space $\mathcal{D}_L^m$ is of prohibitive complexity. Instead, we constrain ourselves to those $s \in \mathcal{D}_L^m$ for which argument in (1) is small. [Note that these are the signal vectors whose contribution to the numerator and denominator in (3) is significant.]

Assume that the search in (2) yields the set of points $\mathcal{S} = \{s^{(1)}, s^{(2)}, \ldots, s^{(l)}\}$. The vector $s \in \mathcal{S}$ that minimizes (1) is the solution to the MAP detection problem. The soft information for each bit $c_i$ can be estimated from (3), by only summing the terms in the numerator and denominator such that $s \in \mathcal{S}$.

Figure 1 shows the bit-error performance of the system with 4 transmit and 4 receive antennas, $16 - QAM$ constellation and parallel concatenated turbo code with rate $R = 1/2$ and length 9216 information bits. For each iteration of the FP-MAP, turbo (inner) decoder performs 8 iterations of its own.

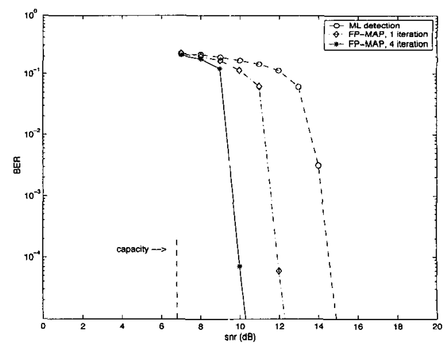


Figure 1: *Rate* 1/2 *turbo code*, $4 \times 4$ *system*, 16-$QAM$

## Acknowledgments

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## References

[1] U. Fincke and M. Pohst, "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis," Mathematics of Computation, vol. 44, pp. 463-471, 1985.

[2] B. Hassibi and H. Vikalo, "Expected complexity of the sphere decoder algorithm," submitted to the IEEE Transactions on Signal Processing, 2002.



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